

# Math 252 Quiz #1

Sept 5, 2013

Due Sept 12, 2013

Name: \_\_\_\_\_  
SID: \_\_\_\_\_

Partner(s): \_\_\_\_\_

**Instructions:** You may work in a group, but you must write you own solutions to the problems and write the names of your collaborators on this worksheet. You may **NOT** get help from a tutor. You must turn in a copy of the questions along with your work which needs to be neat and legible. All numerical answers **MUST** be exact; e.g., you should write  $\pi$  instead of 3.14...,  $\sqrt{2}$  instead of 1.414..., and  $\frac{1}{3}$  instead of 0.3333... All questions will be graded on a yes/no grade scale, and every part is of equal value.

**Show ALL of your work, and justify all answers! No work, no credit!**

**Question 1.** Let  $\mathbf{u} = \langle 2, -4 \rangle$ ,  $\mathbf{v} = \langle -1, 5 \rangle$ ,  $\mathbf{w} = 4\mathbf{i} + 10\mathbf{j}$ ,  
 $\mathbf{a} = \langle -2, 5, 7 \rangle$ ,  $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ ,  $\mathbf{c} = \langle 3, -4, 1 \rangle$

- a) Draw  $\mathbf{u}$ ,  $\mathbf{v}$ ,  $\mathbf{w}$  on the same set of axes.
- b) Draw  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$  on the same set of axes.
- c) Find the following, write both in standard and component form.
  - i)  $\mathbf{u} + \mathbf{v}$
  - ii)  $2\mathbf{u} + \mathbf{v} - 4\mathbf{w}$
  - iii)  $\mathbf{a} - \mathbf{b}$
  - iv)  $2\mathbf{b} - 2\mathbf{c} + \mathbf{a}$
- d) Find  $\mathbf{u} \cdot \mathbf{v}$
- e) Find  $\mathbf{u} \cdot \mathbf{w}$
- f) Find  $\|\mathbf{u}\|$
- g) Find  $\|\mathbf{b}\|$
- h) Find the norm of  $\mathbf{w}$ 
  - i) Find the norm of  $\mathbf{a}$
  - j) Find  $\|\mathbf{v}\|^2$
  - k) Find  $\|\mathbf{c}\|^2$ 
    - l) Find the length of  $\mathbf{b}$
- m) Find the length of  $\mathbf{u}$
- n) Normalize the vectors  $\mathbf{u}$ ,  $\mathbf{v}$ ,  $\mathbf{w}$ ,  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$ 
  - o) Find the angle between  $\mathbf{u}$  and  $\mathbf{w}$
  - p) Find the angle between  $\mathbf{a}$  and  $\mathbf{b}$
  - q) Find the projection of  $\mathbf{u}$  onto  $\mathbf{v}$
  - r) Find the projection of  $\mathbf{a}$  onto  $\mathbf{c}$
  - s) Find the cross product of  $\mathbf{a} \times \mathbf{b}$
  - t) Find the cross product of  $\mathbf{b} \times \mathbf{c}$
  - u) Find the cross product of  $\mathbf{u} \times \mathbf{w}$
  - v) Find the parametric equations for the line between  $\mathbf{u}(2, -4)$  and  $\mathbf{w}(4, 10)$
  - w) Find the parametric equations for the line between  $\mathbf{a}(-2, 5, 7)$  and  $\mathbf{c}(3, -4, 1)$
  - x) Find the area of the triangle consisting of the points  $\mathbf{u}(2, -4)$ ,  $\mathbf{v}(-1, 5)$ , and  $\mathbf{w}(4, 10)$
  - y) Find the area of the parallelogram spanned by the vectors  $\mathbf{a}$  and  $\mathbf{b}$
  - z) Find the volume of the parallelepiped spanned by the vectors  $\mathbf{a}$ ,  $\mathbf{b}$ , and  $\mathbf{c}$

**Question 2.** Given the points  $P(2, 3)$ ,  $Q(8, 6)$ ,  $R(-4, 6)$   
 $A(1, 1, 1)$ ,  $B(-2, 6, 3)$ ,  $C(2, 3, 1)$

- a) Do the points  $P$ ,  $Q$ ,  $R$  lie on the same line? Why/why not?
- b) Do the points  $A$ ,  $B$ ,  $C$  lie on the same line? Why/why not?
- c) Find the line passing through the points  $P$  and  $R$
- d) Find the line passing through the points  $A$  and  $B$
- e) Find the line passing through the point  $C$  in the direction  $\mathbf{d}=2\mathbf{j}-\mathbf{k}$

**Question 3.** Are the following lines intersecting, parallel, or skewed?

- a)  $\ell_1=(-6t, 1+9t, -3t)$ ,  $\ell_2=(1+2t, 4-3t, t)$
- b)  $\ell_1=(3-4t, 2-3t, 1+2t)$ ,  $\ell_2=(t, 1+2t, 2+3t)$
- c)  $\ell_1=(1+2t, 3t, 2-t)$ ,  $\ell_2=(-1+t, 4-3t, t)$

**Question 4.** Find a unit vector orthogonal to  $\langle 2, 1, -3 \rangle$  and is in the  $xz$  plane.

**Question 5.** Given  $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$  are vectors in  $\mathbb{R}^3$ , are the following meaningful? why or why not?

- a)  $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$
- b)  $\mathbf{a} \times (\mathbf{b} \cdot \mathbf{c})$
- c)  $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$
- d)  $(\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$
- e)  $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d})$
- f)  $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$

**Question 6.** Find the determinants of the the following.

a)  $\begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix}$

b)  $\begin{vmatrix} 3 & -5 & 1 \\ 0 & 2 & -2 \\ 3 & 1 & 1 \end{vmatrix}$

c)  $\begin{vmatrix} 1 & 1 & 1 \\ 2 & -2 & -3 \\ -3 & 1 & 2 \end{vmatrix}$

**Question 7.** describe all unit vectors orthogonal to the given vectors.

- a)  $\mathbf{i}, \mathbf{j}$
- b)  $\mathbf{k}, \mathbf{i}$
- c)  $\mathbf{j}, \mathbf{k}$
- d)  $5\mathbf{i}+2\mathbf{j}-1\mathbf{k}, -2\mathbf{i}-\mathbf{j}+7\mathbf{k}$

**Question 8.** Find the equation for the plane that

- a) perpendicular to  $\mathbf{v}=\langle 2, 1, -3 \rangle$  and passes through  $(6, 6, 6)$
- b) perpendicular to the line  $\ell= \langle 2, 4, 1 \rangle t+ (6, 6, 6)$  and passes through  $(8, 1, 0)$
- c) passes through the points  $(2, -4, 6), (1, 3, 9), (-3, 6, 1)$
- d) passes through the points  $(3, 5, -1), (4, -2, 3)$  and is parallel to the line  $\mathbf{v}= \langle 3, -1, 4 \rangle t+ (6, 6, 6)$

**Question 9.** Find the distance between plane and point

- a) point  $(3, 2, -1)$  and the plane  $x + 2y + 2z + 5 = 0$
- b) point  $(-2, 3, 9)$  and the plane passing through the origin that is perpendicular to  $\mathbf{i}+2\mathbf{j}+\mathbf{k}$

**Question 10.** For each of the following

- a)  $\mathbf{c}(t)=2\cos t \mathbf{i} + 2\sin t \mathbf{j} + t\mathbf{k}$  at  $t = \frac{3}{2}\pi$
- b)  $\mathbf{r}(t)=t\mathbf{i}+ t^2\mathbf{j}+3/2 \mathbf{k}$  at  $t = 2$ 
  - i) sketch the curve ... yea that's right.
  - ii) find the velocity vector at the given point.
  - iii) find the tangent line at the given point.
  - iv) If the particle flies off the curve on a tangent at the given point find the position of the particle when  $t = 5$ ?