Math 252 Quiz #2

Oct 3rd, 2013

Due Oct 10th, 2013 at 6:56PM

Name:_____ **Partner(s):**_____
SID:_____

Instructions: You may work in a group, but you must write you own solutions to the problems and write the names of your collaborators on this worksheet. You may **NOT** get help from a tutor. You must turn in a copy of the questions along with your work which needs to be neat and legible. Your work must be stapled. All numerical answers **MUST** be exact; e.g., you should write π instead of 3.14..., $\sqrt{2}$ instead of 1.414..., and $\frac{1}{3}$ instead of 0.3333... All questions will be graded on a yes/no grade scale, and every part is of equal value.

Show ALL of your work, and justify all answers! No work, no credit!

Question 1. Compute the gradient of the given functions.

a) $f(x,y) = \sqrt{x^2+y^2}$

b) $f(x,y) = \arctan(xy)$

c) $f(x,y,z) = xy^2 + yz^2 + zy^2$

d) $f(x,y,z) = xe^y + ye^z + ze^x$

e) $f(x,y,z) = (x+2y+3z)^{3/2}$

Question 2. Sketch the vector field.

a) $\mathbf{F}(x,y) = \mathbf{i} + x\mathbf{j}$

b) $\mathbf{F}(x,y) = y\mathbf{i} + \frac{1}{2}\mathbf{j}$

c) $\mathbf{F}(x,y) = x\mathbf{i} - y^2\mathbf{j}$

Question 3. Sketch the gradient field of each of the following

a) $f(x,y) = x^2 + y^2$

b) $f(x,y) = x + y$

c) $f(x,y) = x - y^2$

d) $f(x,y) = \left(x^2 - y^2\right)/4$

e) $f(x,y) = \sqrt{x^2+y^2}$

Question 4. Compute the directional derivative of each function at the given point in the direction of the unit vector parallel to the given vector

a) $f(x,y) = \arctan(xy)\ ;\ (x_0,y_0) = (1,2)\ ;\ \mathbf{v} = \langle 5,10\rangle$

b) $f(x,y) = 1 + 2x\sqrt{y}\ ;\ (x_0,y_0) = (3,4)\,;\mathbf{v} = \langle 4,-3\rangle$

c) $f(x,y,z) = e^{x^2+y^2+z^2}; (x_0,y_0,z_0) = (1,10,100)\,;\mathbf{v} = \langle 1,-1,-1\rangle$

d) $f(x,y,z) = xyz\ ;(x_0,y_0,z_0) = (1,0,1)\,;\mathbf{v} = \langle 1,0,-1\rangle$

e) $f(x,y,z) = xy^2 + yz^2 + zy^2; (x_0,y_0,z_0) = (1,1,1)\,;\mathbf{v} = \langle 2,0,-1\rangle$

Question 5. Find the maximum rate of change of f at the given point; and the direction in which it occurs.

a) $f(x,y) = y^2/x, (2,4)$

b) $f(x,y) = sin\,(x,y)\,, (1,0)$

c) $f(x,y,z) = \sqrt{x^2+y^2+z^2}, (3,6,-2)$

d) $f(x,y,z) = tan\,(x+2y+3z)\,, (-5,1,1)$

Question 6. The temperature T in a metal ball is inversely proportional to the distance from the center of the ball, which we take to be the origin. The temperature at the point $(1, 2, 2)$ is $120°$.

a) Find the rate of change of T at $(1, 2, 2)$ in the direction toward the point $(2, 1, 3)$

b) Show that for any point in the ball, the direction of greatest increase in temperature is given by a vector that points toward the origin.

Hint: What is the distance formula, and what does it mean to be inversely proportional?

Question 7. Find the equation for the tangent plane to the surface at the indicated point.

a) $x^2 + 2y^2 + 3z^2 = 10$; $\left(1, \sqrt{3}, 1\right)$

b) $y = x^2 - z^2$; $(4, 7, 3)$

c) $x^2 - 3y^2 + z^2 + yz = 2$; $(2, 1, -1)$

Question 8. Find $\partial y/\partial x$.

a) $x^2 + y^4 = 2$

b) $cos\,(x - y) = xe^y$

Question 9. Determine near what points the surface $F(x, y, z) = 0$ can be described as the graph of a differentiable function $z = f(x, y)$.

a) $x^2 + xyz + 2z^3 = 0$

b) $z^2x^3 + xy^2 - 4 = 0$

Question 10. Find all second partial derivatives of the following.

a) $f(x, y) = 2xy/\left(x^2 + y^2\right)^2 ; (x, y) \neq (0, 0)$

b) $f(x, y) = x^3y^5 + 2x^4y$

c) $f(x, y) = ln\left(x^2 + y^2\right)$

d) $f(x, y) = xye^y$

Question 11. Find the indicated partial derivatives.

a) $f(x, y) = 3xy^4 + x^3y^2$; f_{xxy}, f_{yxyx}

b) $f(x, y) = x^2e^{-cy}$; f_{yyy}, f_{yyx}

c) $f(x, y) = x^3 sin\,(x + 2y)$; $\frac{\partial^3 f}{\partial x \partial y \partial x}, \frac{\partial^3 f}{\partial y \partial y \partial x}, \frac{\partial^3 f}{\partial x^3}$

Question 12. Determine the second order Taylor formula for $f(x, y) = e^{(x-1)^2} cos\,(y)$ at the point $(0, 0)$.

Question 13. Find the local maximum and minimum values and saddle point(s) of the following functions.

a) $f(x,y) = 9 - 2x + 4y - x^2 - 4y^2$

b) $f(x,y) = 3 + 2x^2 - xy + y^2$

c) $f(x,y) = ycos(x)$

d) $f(x,y) = e^y\left(y^2 - x^2\right)$

Question 14. Find the absolute maximum and minimum values of f on the set D.

a) $f(x,y) = 1 + 4x - 5y$, D is the closed triangular region with vertices $(0,0), (2,0), (0,3)$.

b) $f(x,y) = 3 + xy - x - 2y$, D is the closed triangular region with vertices $(1,0), (5,0), (1,4)$.

c) $f(x,y) = 4x + 6y - x^2 - y^2$, $D = \{(x,y) \mid 0 \leq x \leq 4, 0 \leq y \leq 5\}$.