

Math 252 Quiz #3

Oct 10th, 2013

Due Oct 15th, 2013 at 6:55PM

Name: _____
SID: _____

Partner(s): _____

Instructions: You may work in a group, but you must write you own solutions to the problems and write the names of your collaborators on this worksheet. You may **NOT** get help from a tutor. You must turn in a copy of the questions along with your work which needs to be neat and legible. Your work must be stapled. All numerical answers **MUST** be exact; e.g., you should write π instead of 3.14..., $\sqrt{2}$ instead of 1.414..., and $\frac{1}{3}$ instead of 0.3333... All questions will be graded on a yes/no grade scale, and every part is of equal value.

Show ALL of your work, and justify all answers! No work, no credit!

Question 1. Find the extrema of f subject to the stated constraints.

- a) $f(x, y) = x^2 + y^2; xy = 1$
- b) $f(x, y) = x^2y; x^2 + 2y^2 = 6$
- c) $f(x, y, z) = 8x - 4z; x^2 + 10y^2 + z^2 = 5$
- d) $f(x, y, z) = xyz; x^2 + 2y^2 + z^2 = 6$

Question 2. Find the speed and acceleration of the given path.

- a) $\mathbf{c}(t) = 6t\mathbf{i} + 3t^2\mathbf{j} + t^3\mathbf{k}$
- b) $\mathbf{c}(t) = \langle \sin 3t, \cos 3t, 2t^{3/2} \rangle$

Question 3. Find the arc length of the given curve on the specified interval.

- a) $\mathbf{c}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + t\mathbf{k}; 0 \leq t \leq 4\pi$
- b) $\mathbf{c}(t) = \left\langle t + 1, \frac{2\sqrt{2}}{3}t^{3/2} + 7, \frac{1}{2}t^2 \right\rangle; 1 \leq t \leq 2$

Question 4. Sketch the given vector field

- a) $\mathbf{F}(x, y) = \langle 4, 0 \rangle$
- b) $\mathbf{F}(x, y) = \langle x, y \rangle$
- c) $\mathbf{F}(x, y) = \langle y, -2x \rangle$

Question 5. Find the divergence and curl of the following vector fields.

- a) $\mathbf{F}(x, y, z) = xyz\mathbf{i} - x^2y\mathbf{k}$
- b) $\mathbf{F}(x, y, z) = x^2yz\mathbf{i} + xy^2z\mathbf{j} + xyz^2\mathbf{k}$
- c) $\mathbf{F}(x, y, z) = xyz\mathbf{i} - x^2y\mathbf{k}$
- d) $\mathbf{F}(x, y, z) = \langle \ln x, \ln(xy), \ln(xyz) \rangle$

Question 6. let f be a scalar field and \mathbf{F} a vector field. State whether each expression is meaningful. If not explain why. If so, state whether it is a scalar field or a vector field.

- a) $\text{curl} f$
- b) $\text{grad } f$
- c) $\text{div } \mathbf{F}$
- d) $\text{curl grad } f$
- e) $\text{grad } \mathbf{F}$
- f) $\text{grad div } \mathbf{F}$
- g) $\text{div grad } f$
- h) $\text{grad div } f$
- i) $\text{curl curl } \mathbf{F}$
- j) $\text{div div } \mathbf{F}$
- k) $\text{grad } f \times \text{div } \mathbf{F}$
- l) $\text{div curl grad } f$

Question 7. Determine whether or not the vector field is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$

- a) $\mathbf{F}(x, y, z) = y^2 z^3 \mathbf{i} + 2xyz^3 \mathbf{j} + 3xy^2 z^2 \mathbf{k}$
- b) $\mathbf{F}(x, y, z) = 2xy \mathbf{i} + (x^2 + 2yz) \mathbf{j} + y^2 \mathbf{k}$
- c) $\mathbf{F}(x, y, z) = e^z \mathbf{i} + \mathbf{j} + xe^z \mathbf{k}$
- d) $\mathbf{F}(x, y, z) = y \cos(xy) \mathbf{i} + x \cos(xy) \mathbf{j} - \sin(z) \mathbf{k}$