Math 252 Quiz #4

Oct 24th, 2013

Due Oct 31th, 2013 at 6:56PM

Name:______________________ **Partner(s):**______________________

SID:______________________

Instructions: You may work in a group, but you must write you own solutions to the problems and write the names of your collaborators on this worksheet. You may **NOT** get help from a tutor. You must turn in a copy of the questions along with your work which needs to be neat and legible. Your work must be stapled. All numerical answers **MUST** be exact; e.g., you should write π instead of 3.14..., $\sqrt{2}$ instead of 1.414..., and $\frac{1}{3}$ instead of 0.3333... All questions will be graded on a yes/no grade scale, and every part is of equal value.

Show ALL of your work, and justify all answers! No work, no credit!

Question 1. Calculate the iterated integral

a) $\int\limits_0^2 \int\limits_0^{\pi/2} x\sin(y)\ dydx$

b) $\int\limits_0^1 \int\limits_0^1 xy\sqrt{x^2+y^2}\, dydx$

c) $\int\limits_1^2 \int\limits_1^4 \left(\frac{x}{y}+\frac{y}{x}\right)\ dxdy$

d) $\int\limits_0^2 \int\limits_0^{\pi} r\sin^2\theta\, d\theta dr$

Question 2. Calculate the double integral

a) $\iint\limits_R \left(6x^2y^3-5y^2\right)\ dA$ with $R=\{(x,y)\,|0\le x\le 3, 0\le y\le 1\}$

b) $\iint\limits_R \frac{1+x^2}{1+y^2}\, dA$ with $R=\{(x,y)\,|0\le x\le 1, 0\le y\le 1\}$

c) $\iint\limits_R xye^{x^2y}\, dA$ with $R=\{(x,y)\,|0\le x\le 1, 0\le y\le 2\}$

d) $\iint\limits_R \sqrt{64-x^3}\, dA$ with $R=\{(x,y)\,|-4\le x\le 4, 0\le y\le 4\}$

Question 3. Find the volume of the solid enclosed by the surface

a) $z=1+e^x\sin(y)$ and the planes $x=\pm1, y=0, y=\pi$ and $z=0$

b) $z=x\sec^2(y)$ and the planes $z=0, x=0, x=2, y=0$, and $y=\pi/4$

Question 4. Evaluate the iterated integral

a) $\int\limits_0^1 \int\limits_0^{\sqrt{y}} xy^2\, dxdy$

b) $\int\limits_0^2 \int\limits_y^{2y} xy\, dxdy$

c) $\int\limits_0^1 \int\limits_0^{y} \sqrt{1-y^2}\, dxdy$

d) $\int\limits_0^1 \int\limits_{\sqrt{x}}^{2} \frac{1}{1+y^2}\, dydx$

e) $\int\limits_0^8 \int\limits_{\sqrt[3]{y}}^{2} e^{x^4}\, dxdy$

Question 5. Evaluate the double integral

a) $\iint\limits_D y^2\, dA$ with $D = \{(x, y) \,|\, -y-2 \le x \le y, -1 \le y \le 1\}$

b) $\iint\limits_D x\sqrt{y^2 - x^2}\, dA$ with $D = \{(x, y) \,|0 \le x \le y, 0 \le y \le 1\}$

c) $\iint\limits_D x^3\, dA$ with $D = \{(x, y) \,|1 \le x \le e, 0 \le y \le lnx\}$

d) $\iint\limits_D (2x - y)\; dA$ with D is bounded by the circle with center the origin and radius 2.

e) $\iint\limits_D 2xy\, dA$ with D is the triangular region with vertices $(0, 0)\,,(1, 2)\,(0, 3)$

Question 6. Find the volume of the given solid

a) under the plane $x + 2y - z = -$ and above the region bounded by $y = x$ and $y = x^4$

b) under the surface $z = 2x + y^2$ and above the region bounded by $x = y^2$ and x^3

c) enclosed by the paraboloid $z = x^2 + 3y^2$ and the planes $x = 0, y = 1, y = x, z = 0$

d) bounded by the cylinders $x^2 + y^2 = r^2$ and $y^2 + z^2 = r^2$

e) bounded by the cylinder $x^2 + y^2 = 1$ and the planes $y = z, x = 0.z = 0$ in the first octant.

Question 7. Sketch the region of integration and change the order of integration

a) $\int\limits_1^4 \int\limits_0^{\sqrt{x}} f(x, y)\; dydx$

b) $\int\limits_0^3 \int\limits_{-\sqrt{9-y^2}}^{\sqrt{9+y^2}} f(x, y)\; dxdy$

c) $\int\limits_1^2 \int\limits_0^{lnx} f(x, y)\; dydx$

d) $\int\limits_0^1 \int\limits_{arctanx}^{\pi/4} f(x, y)\; dydx$

Question 8. Evaluate the given integral by changing to polar coordinates

a) $\iint\limits_D xy\, dA$, D is the disk with center the origin and radius 3

b) $\iint\limits_D (x + y)\; dA$, D is the region that lies to the left of the y-axis between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$

c) $\iint\limits_D e^{-x^2-y^2}\; dA$, D is the region bounded by the semicircle $\sqrt{4 - y^2}$

Question 9. Evaluate the iterated integral

a) $\int_0^1 \int_0^z \int_0^{x+z} 6xz\, dydxdz$

b) $\int_0^{\sqrt{\pi}} \int_0^x \int_0^{xz} x^2 sin\,(y)\; dydzdx$

c) $\int_0^{\pi/2} \int_0^y \int_0^x cos\,(x+y+z)\; dzdxdy$

Question 10. Evaluate the iterated integral

a) $\iiint_E 2x\, dV$ where $E = \left\{(x,y,z)\,|0 \le x \le \sqrt{4-y^2}, 0 \le y \le 2, 0 \le z \le y\right\}$

b) $\iiint_E 6xy\, dV$ where E lies under the plane $z = 1+x+y$ and above the region in the $xy-$plane bounded by the curves $y = \sqrt{x}, y = 0$ and $x = 1$

Question 11. Evaluate the integral by changing to cylindrical coordinates.

a) $\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{2} xz\, dzdxdy$

b) $\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} \sqrt{x^2+y^2}\, dzdydx$

Question 12. Evaluate the integral by changing to spherical coordinates coordinates.

a) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy\, dzdydx$

b) $\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} z^2\, dzdxdy$

Question 13. Convert the following rectangular coordinates to...

a) cylindrical coordinates

b) spherical coordinates

i) $\left(1, \sqrt{3}, 2\sqrt{3}\right)$

ii) $\left(-1, 1, \sqrt{6}\right)$

iii) $(0, -1, 1)$