Math 252 Quiz #5

Dec 3rd, 2013

Due December 12th, 2013 at 6:56PM

Name:______________________ **Partner(s):**______________________

SID:______________________

Instructions: You may work in a group, but you must write you own solutions to the problems and write the names of your collaborators on this worksheet. You may **NOT** get help from a tutor. You must turn in a copy of the questions along with your work which needs to be neat and legible. Your work must be stapled. All numerical answers **MUST** be exact; e.g., you should write π instead of 3.14..., $\sqrt{2}$ instead of 1.414..., and $\frac{1}{3}$ instead of 0.3333... All questions will be graded on a yes/no grade scale, and every part is of equal value.

Show ALL of your work, and justify all answers! No work, no credit!

Question 1. use Green's theorem to evaluate the line integral.

a) $\int\limits_C 2xy\,dx + (x+y)\,dy$
C: boundary of the region lying between the graphs of $y = 0$ and $y = 4 - x^2$

b) $\int\limits_C y^2\,dx + xy\,dy$
C: boundary of the region lying between the graphs of $y = 0$, $y = \sqrt{x}$, and $x = 9$

c) $\int\limits_C \left(x^2 - y^2\right)\,dx + 2xy\,dy$
$C: x^2 + y^2 = a^2$

d) $\int\limits_C \left(x^2 - y^2\right)\,dx + 2xy\,dy$
$C: r = 1 + cos\theta$

e) $\int\limits_C 2\arctan\frac{x}{y}\,dx + ln\left(x^2 + y^2\right)\,dy$
$C: x = 4 + 2\cos\theta, y = 4 + \sin\theta$

f) $\int\limits_C e^x\cos(2y)\,dx - 2e^x\sin(2y)\,dy$
$C: x^2 + y^2 = a^2$

g) $\int\limits_C \sin x\cos x\,dx + (xy + \cos x\sin y)\,dy$
C: boundary of the region lying between the graphs of $y = x$ and $y = \sqrt{x}$

h) $\int\limits_C xy\,dx + (x+y)\,dy$
C: boundary of the region lying between the graphs of $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$

i) $\int\limits_C 3x^2e^x\,dx + e^y\,dy$
C: boundary of the region lying between the graphs of $y = 0$ and $y = 4 - x^2$

j) $\int\limits_C 2xy\,dx + (x+y)\,dy$
C : boundary of the region lying between the squares with vertices $(1,1), (-1,1), (-1,-1)$, and $(1,-1)$ and $(2,2), (-2,2), (-2,-2), and\ (2,-2)$

Question 2. Use Green's Theorem to calculate the work done by the force $\mathbf{F}$ on a particle that is moving counterclockwise around the closed path C

a) $\mathbf{F}(x,y) = xy\mathbf{i} + (x+y)\mathbf{j}$
$C: x^2 + y^2 = 4$

b) $\mathbf{F}(x,y) = (e^x - 3y)\mathbf{i} + (e^y + 6x)\mathbf{j}$
$C: r = 2\cos\theta$

c) $\mathbf{F}(x,y) = \left(x^{3/2} - 3y\right)\mathbf{i} + \left(6x + 5\sqrt{y}\right)\mathbf{j}$
C: boundary of the triangle with vertices $(0,0), (5,0)$, and $(0,5)$

d) $\mathbf{F}(x,y) = \left(3x^2 + y\right)\mathbf{i} + 4xy^2\mathbf{j}$
C: is the boundary of the region lying between the graphs of $y = \sqrt{x}, y = 0,$ and$x = 9$

Question 3. Use a line integral to find the area of the region inside the loop of the folium of Descartes bounded by the graph of $x = \frac{3t}{t^3+1}, y = \frac{3t^2}{t^3+1}$

Question 4. Use Stokes's theorem to evaluate $\int\limits_C \mathbf{F}\cdot d\mathbf{r}$

a) $\mathbf{F}(x,y,z) = 2y\mathbf{i} + 3z\mathbf{j} + x\mathbf{k}$
C: triangle with vertices $(0,0,0), (0,2,0), (1,1,1)$

b) $\mathbf{F}(x,y,z) = z^2\mathbf{i} + x^2\mathbf{j} + y^2\mathbf{k}$
$S: z = 4 - x^2 - y^2, z \geq 0$

c) $\mathbf{F}(x,y,z) = 3xz\mathbf{i} + y\mathbf{j} + 4xy\mathbf{k}$
$S: z = 9 - x^2 - y^2, z \geq 0$

d) $\mathbf{F}(x,y,z) = z^2\mathbf{i} + y\mathbf{j} + xz\mathbf{k}$
$S: \sqrt{4 - x^2 - y^2}$

e) $\mathbf{F}(x,y,z) = x^2\mathbf{i} + z^2\mathbf{j} - xyz\mathbf{k}$
$S: \sqrt{4 - x^2 - y^2}$

Question 5. use the Divergence theorem to evaluate $\iint\limits_S \mathbf{F}\cdot\mathbf{N}\, d\mathbf{S}$

a) $\mathbf{F}(x,y,z) = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$
$S: x = 0, x = a, y = 0, y = a, z = 0, z = a$

b) $\mathbf{F}(x,y,z) = x^2z^2\mathbf{i} - 2y\mathbf{j} + 3xyz\mathbf{k}$
$S: x = 0, x = a, y = 0, y = a, z = 0, z = a$

c) $\mathbf{F}(x,y,z) = x^2\mathbf{i} - 2xy\mathbf{j} + xyz\mathbf{k}$
$S: z = \sqrt{a^2 - x^2 - y^2}, z = 0$

d) $\mathbf{F}(x,y,z) = xyz\mathbf{j}$
$S: x^2 + y^2 = 9, z = 0, z = 4$

e) $\mathbf{F}(x,y,z) = xy\mathbf{i} + 4y\mathbf{j} + xz\mathbf{k}$
$S: x^2 + y^2 + z^2 = 9$

f) $\mathbf{F}(x,y,z) = xe^z\mathbf{i} + ye^z\mathbf{j} + e^z\mathbf{k}$
$S: z = 4 - y, z = 0, x = 0, x = 6, y = 0$