Last name: *First name:*

Recommended problems - *Please do NOT turn these in*:

- §4.1: 1c, 9c, 11c, 15, 19, 23, 37.
- §4.2: 3, 5b, 7, 9b, 11b, 13a, 17b, 29, 31.
- §4.3: 1ac, 3, 5, 7ac, 9, 11b, 13, 15, 25

Submitted problems: *Please turn these problems in*:

(1) Prove the identity $\|\mathbf{u}+\mathbf{v}\|^2 + \|\mathbf{u}-\mathbf{v}\|^2 = 2(\|\mathbf{u}\|^2 + \|\mathbf{v}\|^2)$.

(2) Show that if $\mathbf{u}$ is orthogonal to both $\mathbf{v}$ and $\mathbf{w}$, then $\mathbf{u}$ is orthogonal to $k_1\mathbf{v} + k_2\mathbf{w}$ for all scalar k_1 and k_2.

(3) Determine if $T(x_1, x_2) = (2x_1, 3x_2, x_1 + x_2 + 1)$ is a linear transformation. Justify your answer.

(4) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be defined as $T(x, y, z) = (3x - y, 2y + z)$.
   (a) Prove that T a linear transformation.
   (b) Find the standard matrix A for the linear transformation T
   (c) Use the matrix A in part (b) to compute $T(\mathbf{x})$ where $\mathbf{x} = \begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$.

(5) Find the standard matrix for the following operators on $\mathbb{R}^3$
   (a) T_1 is an orthogonal projection on the $yz-$plane
   (b) T_2 is a reflection about the $xy-$plane
   (c) T_3 is a rotation of 60° about the $z-$axis.
   (d) $T_3 \circ T_1 \circ T_2$. Describe in words the affect that this composition of linear operators has on vectors in $\mathbb{R}^3$.

(6) Show that the range of the operator $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$\begin{aligned} w_1 &= x_1 - 2x_2 + x_3 \\ w_2 &= 5x_1 - x_2 + 3x_3 \\ w_3 &= 4x_1 + x_2 + 2x_3 \end{aligned}$$

is not all of $\mathbb{R}^3$. Give an example of a vector that is not in the range.