Last name: *First name:*

Recommended problems - *Please do NOT turn these in*:

- §5.2: 1acd, 3ac, 5ab, 7, 11, 13, 23.
- §5.3: 1, 3, 7, 11, 13, 15, 21, 25.

Submitted problems: *Please turn these problems in*:

(1) Consider the vectors $\mathbf{v_1} = (2, -1, 3)$, $\mathbf{v_2} = (4, 1, 2)$, and $\mathbf{v_3} = (8, -1, 8)$.
  (a) Is $\mathbf{x} = (2, 0, -5)$ a linear combination of $\mathbf{v_1},\ \mathbf{v_2},\ \mathbf{v_3}$? If yes, write down the linear combination. If no, explain why not.
  (b) Determine whether the given vectors span $\mathbb{R}^3$. Clearly justify your answer.

(2) Consider the vector space $\mathbb{R}^3$. Let $V = \{(1, 2, -1),\ (1, 1, 0),\ (1, 0, 1)\}$.
  (a) Is $w = (2, 1, 1)$ in $span(V)$? Explain.
  (b) Find $span(V)$. Does V spans $\mathbb{R}^3$? Explain.

(3) State (with brief explanation) whether the following statements are true or false.
  (a) All vectors of the form $(a, 0, -a)$ form a subspace of $\mathbb{R}^3$.
  (b) Every set of vectors spanning $M_{2,3}$ contains at least 6 vectors.
  (c) If $span(S_1) = span(S_2)$, then $S_1 = S_2$.

(4) For what value(s) of λ is the set of vectors $(\lambda^2 - 5, 1, 0)$, $(2, -2, 3)$, $(2, 3, -3)$ linearly independent?

(5) Let the set $\{\mathbf{v}_1, \mathbf{v}_2\}$ be linearly dependent. Prove that $\{\mathbf{v}_1 + 2\mathbf{v}_2, 3\mathbf{v}_1 - \mathbf{v}_2\}$ is also linearly dependent.

(6) State (with brief explanation) whether the following statements are true or false.
  (a) Every set of vectors in $\mathbb{R}^3$ containing two vectors is linearly independent.
  (b) If $\{\mathbf{v_1},\ \mathbf{v_2}\}$ is a linearly dependent set, then each vector is a scalar multiple of the other.
  (c) If $\{\mathbf{v_1},\ \mathbf{v_2},\ \mathbf{v_3}\}$ is a linearly indepenent set, then so is the set $\{k\mathbf{v_1},\ k\mathbf{v_2},\ k\mathbf{v_3}\}$ for any nonzero scalar k.