

Last name:

First name:

Recommended problems - *Please do NOT turn these in:*

- §6.1: 1, 3a, 9, 13, 15, 21, 27b
- §6.2: 3, 5e, 8, 13, 17, 21, 23, 33, 37
- §6.3: 3, 5, 7b, 9, 11, 13, 17a, 19, 29, 30, 35

Submitted problems: Please turn these problems in:

- (1) Let $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$. Define

$$\langle \mathbf{u}, \mathbf{v} \rangle = \max\{|u_1v_1|, |u_2v_2|, |u_3v_3|\}$$

Determine if $\langle \mathbf{u}, \mathbf{v} \rangle$ is an inner product on \mathbb{R}^3 . If it is, verify that the inner product axioms hold. If it is not, list the axioms that do not hold and provide concrete example(s) to demonstrate that the axiom fails.

- (2) Let $M_{2,2}$ has the inner product as in Example 7 of the text (§6.1, page 301).

Let $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 \\ -2 & 4 \end{bmatrix}$. Compute $d(A, B)$.

- (3) Problem 16be, §6.1, page 305.

- (4) Let the vector space P_2 has the inner product

$$\langle p(x), q(x) \rangle = \int_0^1 p(x)q(x)dx$$

- (a) Compute $\|p(x)\|$ where $p(x) = \sin x$
- (b) Find the angle between $p(x) = \sin(\pi x)$ and $q(x) = \cos(\pi x)$.
- (c) Let $S = \{1, x, x^2\}$ be the standard basis for P_2 . Use the Gram-Schmidt process to transform S into an orthonormal basis relative to the given inner product.

- (5) Let V be the subspace of \mathbb{R}^4 spanned by the vectors

$$\mathbf{v}_1 = (1, 1, -1, -1), \mathbf{v}_2 = (-1, 0, 1, 0), \mathbf{v}_3 = (0, 1, 0, -3), \mathbf{v}_4 = (0, 1, 0, 1)$$

- (a) Find a basis B for V using given vectors \mathbf{v}_i , $i = 1, 2, 3, 4$.
- (b) Let $\mathbf{u} = (-1, 3, 1, 1)$. Find $[\mathbf{u}]_B$.
- (c) Use the Gram-Schmidt process to transform B into an orthonormal basis B' .
- (d) Find $[\mathbf{u}]_{B'}$ using \mathbf{u} in part (b).