Last name: *First name:*

Recommended problems - *Please do NOT turn these in*:

- §6.5: 1b, 3b, 7, 11abc.
- §7.1: 1ce, 3ce, 5a, 11, 21, 25.
- §7.2: 3, 11, 13, 19, 25.
- §7.3: 1ab, 3ce, 5a, 11, 21, 25.

Submitted problems: *Please turn these problems in*:

(1) Let $S = \{(1,0,0),\ (-1,0,1),(0,1,-1)\}$ and $T = \{(1,1,0),\ (0,1,1),\ (0,0,1)\}$ be bases for $\mathbb{R}^3$.
  (a) Find the transition matrix $P_{T\leftarrow S}$ from S to T.
  (b) Find the transition matrix $P_{S\leftarrow T}$ from T to S.
  (c) Let $\mathbf{v} = (1,-2,3)$. Find $[\mathbf{v}]_S$.
  (d) Let $\mathbf{v} = (1,-2,3)$. Find $[\mathbf{v}]_T$ by using transition matrix found in previous part.

(2) Let $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$. Find the eigenvalues and bases for corresponding eigenspaces of the following matrices:
  (a) A.
  (b) A^T
  (c) A^6

(3) Let A be an $n \times n$ matrix such that $A^2 = A$. Prove that A must has 0 or 1 as an eigenvalue.

(4) Let $\mathbf{A} = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$. Determined whether A is diagonalizable. If so, find a matrix P that diagonalizes A and determine $P^{-1}AP$.

(5) Let $\mathbf{A} = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$. Find a matrix P that orthogonally diagonalizes A and use it to compute $\mathbf{A}^5$.