Last name: *First name:*

Recommended problems - *Please do NOT turn these in*:

- §8.1: 3, 9a, 15, 19, 25.
- §8.2: 1, 5, 7a, 8a, 11, 15.
- §8.3: 1e, 3a, 7, 15, 17a.

Submitted problems: *Please turn these problems in*:

(1) Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be a linear transformation defined by

$$T(x_1, x_2, x_3, x_4) = (4x_1 + x_2 + 5x_3 + 2x_4,\ x_1 + 2x_2 + 3x_3).$$

(a) Find a basis for the kernel and the range of T.
(b) Find the rank and nullity of T.
(c) Is T one-to-one? Explain.
(d) Is the vector $\mathbf{u} = (-1, -1, 1, 0)$ in the $ker(T)$? Explain.

(2) Problem 18 in section 8.2, page 401.

(3) Let $T : \mathbf{V} \to \mathbf{W}$ be a linear transformation. Determine whether the following statement is always true or sometimes false; justify your answer.
"If A is a standard matrix representation of T, then A^2 is a standard matrix representation of $T \circ T$."

(4) Suppose that $\mathbf{V}$ and $\mathbf{W}$ have the same basis $\{\mathbf{u_1},\ \mathbf{u_2}\}$. Let $T : \mathbf{V} \to \mathbf{W}$ be a linear transformation. Give an example of T (not identity transformation) such that:
(a) T is its own inverse.
(b) T equals T^2.