M254 PRACTICE TEST 1

Midterm 1 covers chapters 1, 2 (except 2.4), and 3. Please make sure that you go over homework and examples in lectures. Rest assured that your test is no where near this long.

(1) Determine if the following statements is TRUE or SOMETIMES FALSE. If it is true, prove it. If it is false, give a counter example *or* a brief explanation to justify that it is false.
   (a) Let A be $m \times n$ matrix, then the size of AA^T is $n \times n$.
   (b) For any $m \times n$ matrix A, $A^T A$ and AA^T are always symmetric.
   (c) If A and B are invertible, then $(AB)^{-1} = B^{-1}A^{-1}$.
   (d) Let A and B be square matrices, then $(A + B)(A - B) = A^2 - B^2$.
   (e) Let A and B be square matrices, then $det(A + B) = det(A) + det(B)$.
   (f) A is invertible iff it can be expressed as a product of elementary matrices.
   (g) $det(kA) = kdet(A)$ for any scalar k.

(2) Determine if each of the following statement is TRUE or FALSE. Circle T for true, F for false. You DO NOT need to justify your answers.
Let A and B be $n \times n$ matrices.
   (a) T F $A - A^T$ is always a symmetric matrix.
   (b) T F $Tr(A + B) = Tr(A) + Tr(B)$.
   (c) T F If A is row equivalent to indentity matrix, then A is invertible.
   (d) T F $det(\lambda I - A) = det(A - \lambda I)$
   (e) T F If A is invertible, then A^T is also invertible.

(3) Prove:
   (a) If A is nonsingular and $A^{-1} = A^T$, then $det(A) = \pm 1$.
   (b) A and B are invertible if and only if AB is invertible.
   (c) If $A^2 = A$, then $det(A) = 0$ or $det(A) = 1$.

(4) Consider the system of linear equations

$$
\begin{aligned}
2x_1 - 2x_2 + 2x_3 &= 0 \\
-2x_1 + 5x_2 - 2x_3 &= 1 \\
8x_1 + x_2 + 4x_3 &= -1
\end{aligned}
$$

   (a) Solve the system by Gauss-Jordan Elimination.
   (b) Rewrite the system in form of $Ax = b$. Solve the system by computing $x = A^{-1}b$.

(5) Determine if the following system is consistent or inconsitent? And how many solution does it has?

$$
\begin{aligned}
2x_1 - 3x_2 + 4x_3 - x_4 &= 0 \\
7x_1 + x_2 - 8x_3 + 9x_4 &= 0 \\
2x_1 + 8x_2 + x_3 - x_4 &= 0
\end{aligned}
$$

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(6) Let $\mathbf{A} = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix}$ $\mathbf{C} = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 5 \\ 3 & 2 & 4 \end{bmatrix}$.

Assuming that k_1, k_2, k_3, k_4 are not equal 0.

(a) Check if each of the matrices is invertible. If yes, compute its inverses.
(b) Compute $tr(B^2)$ and $C + C^T$
(c) Compute $|A - \lambda I|$ where λ is a real number. Find λ when $|A - \lambda I| = 0$.

(7) Let $A = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 5 & 2 & 0 \\ -1 & -1 & 0 & -1 \end{bmatrix}$.

(a) Show that A is invertible.
(b) Compute $\left|(3A)^T(A^{-1})^2\right|$

(8) Consider the system $\begin{matrix} x & - & ky & = & 1 \\ kx & + & y & = & -1 \end{matrix}$

(a) Solve by Gauss-Jordan Elimination method.
(b) by rewriting the equation in $Ax = b$ and find the inverse of A.
(c) Determine if the following statement is true or false. *Justify your answer.* "The given system of equations is consistent for all values of k".

(9) Let $\mathbf{u} = (3, -2, 0$, $\mathbf{v} = (1, -1, -2)$ and $\mathbf{w} = (0, 1, 1)$ be vectors in 3-space.

(a) Evaluate the following expression if it is valid. If it is not valid, state why.

$\|\mathbf{u}\|(2\mathbf{v} - \mathbf{w})$, $\mathbf{u} \cdot \mathbf{v}$, $\mathbf{u} \times \mathbf{v}$, $(\mathbf{u} \cdot \mathbf{v})\mathbf{w}$, $(\mathbf{u} \cdot \mathbf{v}) \cdot \mathbf{w}$, $(\mathbf{u} \cdot \mathbf{v}) \times \mathbf{w}$,
$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$, $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$, $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$.

(b) Normalize $\mathbf{w}$.

(10) Let $\mathbf{u} = (1, 3, -2)$ and $\mathbf{v} = (2, -1, 1)$. Compute

(a) Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$.
(b) $proj_{\mathbf{v}}\mathbf{u}$ and the vector component of $\mathbf{u}$ orthogonal to $\mathbf{v}$.

(c) Find all scalars c_1, c_2 and c_3 such that

$$c_1(1, 2, 0) + c_2(2, 1, 1) + c_3(0, 3, 1) = (0, 0, 0)$$

(d) Indicate whether the statement is always true or sometimes false. Justtify your answer with a logical argument or a counter-example. Assuming that $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^n$.

(i) If $\mathbf{u} + \mathbf{v} = \mathbf{u} + \mathbf{w}$, then $\mathbf{v} = \mathbf{w}$.

(ii) $\|\mathbf{u} + \mathbf{v}\| = \|\mathbf{u}\| + \|\mathbf{v}\|$.

(iii) If $\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\|\ \|\mathbf{v}\|$, then $\mathbf{u} \times \mathbf{v} = \mathbf{0}$.

(iv) $\|\mathbf{u} - \mathbf{v}\|^2 \leq \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$.