Test 2 covers chapter 4 and section 5.1-5.3. Please also go over the homework assignments.

(1) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be defined as $T(x, y, z) = (3x - y, 2y + z)$.
 (a) Prove that T a linear transformation.
 (b) Find the standard matrix A for the linear transformation T.
 (c) Is T a one-to-one transformation? Explain.
 (d) Is $(3, 3)$ in the range of T?

(2) Let T be a linear operator on $\mathbb{R}^3$ where T is an orthogonal projection on the $yz-$plane.
 (a) Find the standard matrix for T.
 (b) Is T a one-to-one operator?
 (c) Show that range of T is not $\mathbb{R}^3$. Give example of a vector not in the range of T.
 (d) Find eigenvalues and corresponding eigenvectors for T.

(3) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x_1, x_2, x_3) = (x_1,\ x_1 + x_2,\ x_1 + x_2 + x_3)$.
 (a) Is the range of T all of $\mathbb{R}^3$? Justify your answer.
 (b) Is T a one-to-one operator? If T is one-to-one, find its invert. If not, justify why not.

(4) Let V be the set of real numbers. For any x and y in V, the new operations $\oplus$ and $\odot$ are defined as follow:

$$x \oplus y = 2(x + y) \qquad\qquad c \odot x = c^2 x$$

Is V a vector space under these new operation? Go through all ten axioms and specify if each one pass or fail.

(5) Let V be the set of all $n \times n$ symmetric matrices with standard matrix addition and scalar multiplication. Use the ten vector space axioms to prove that V is a vector space.

(6) Let W be a subspace of $\mathbb{R}^n$ and $S = \{\mathbf{v} \in \mathbb{R}^n \mid \mathbf{w}.\mathbf{v} = 0 \text{ for every } \mathbf{w} \text{ in } W\}$. Prove that S is a subspace of $\mathbb{R}^n$.

(7) Consider the set W of all vectors in $\mathbb{R}^4$ of the form $(a + b, b + c, a - b - 2c, b + c)$. Is W a subspace of $\mathbb{R}^4$? Explain.

(8) Let $\{\mathbf{v}_1, \mathbf{v}_2\}$ spans vector space V. Show that the set of vectors $\{\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_1 - \mathbf{v}_2\}$ also spans V.

(9) Let $\mathbf{v_1} = (1, -2, 0, 1)$, $\mathbf{v_2} = (-1, 3, 1, 4)$, and $\mathbf{v_3} = (0, 1, 4, 1)$. Consider set $W = \{\mathbf{v_1},\ \mathbf{v_2},\ \mathbf{v_3}\}$.
 (a) Determine if W is linearly independent or dependent.
 (b) Is $\mathbf{u} = (1, 1, 1, 1)$ in $span(W)$?
 (c) Find $span(W)$.

(10) State (with brief explanation) whether the following statements are true or false.
 (a) All vectors of the form $(a, 1, -a)$ form a subspace of $\mathbb{R}^3$.
 (b) Let $S = \{\mathbf{v_1},\ \mathbf{v_2},\ \mathbf{v_3},\ \mathbf{v_4}\}$ be set of vectors in $\mathbb{R}^3$. S is linearly dependent.
 (c) Any set contains exactly two vectors in $\mathbb{R}^2$ must be linearly independent.

(11) Prove that for any vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$, the vectors $\mathbf{u} - \mathbf{v}$, $\mathbf{v} - \mathbf{w}$, and $\mathbf{w} - \mathbf{u}$ form a linearly dependent set.