Test 3 covers sections 5.4-5.6, chapter 6, and section 7.1

(1) Determine whether the following statements are true or false. Circle T for true, F for false. You DO NOT need to justify your answers.

(a)	T	F	Every orthonormal set of n vectors in $\mathbb{R}^n$ forms a basis for $\mathbb{R}^n$.
(b)	T	F	If A is not a square matrix, then the row vectors of A must be linearly dependent.
(c)	T	F	If $\mathbf{u}$ is in $row(A)$ and $null(A)$ for $n \times n$ matrix A, then $\mathbf{u} = \mathbf{0}$.
(d)	T	F	Let A be $n \times n$ matrix. A is invertible if and only if $dim(row(A)) = n$.
(e)	T	F	Every orthonormal set of n vectors in $\mathbb{R}^n$ forms a basis for $\mathbb{R}^n$.
(f)	T	F	A and A^T have the same eigenvalues and the same eigenvectors.
(g)	T	F	If the row space equals the column space then $A = A^T$.

(2) Find a basis for the subspace of P_2 spanned by the vectors $-1 + x - 2x^2,\ 3 + 3x + 6x^2,\ 9$.

(3) Let $A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ -3 & 3 & 7 & 1 \\ -1 & 3 & 9 & 3 \\ -5 & 3 & 5 & -7 \end{bmatrix}$

(a) Find bases for $row(A),\ col(A),\ null(A)$
(b) Determine $rank(A)$ and $nullity(A)$. Verify the dimension theorem.
(c) Use the Gram-Schmidt process to transform basis $row(A)$ into an orthonormal basis S.
(d) Let $\mathbf{u} = (1, 0, 1, 1)$, find $[\mathbf{u}]_S$.

(4) (a) Prove that $< \mathbf{u}, \mathbf{v} >= 2u_1v_1 + 3u_2v_2$ is an inner product in $\mathbb{R}^2$.
(b) Let $\mathbf{u} = (-3, 2)$ and $\mathbf{v} = (1, 7)$. Use the inner product in part (a) to compute
- $< \mathbf{u}, \mathbf{v} >$
- $\|\mathbf{u}\|$.
- The cosine of the angle between $\mathbf{u} >$ and $\mathbf{v}$

(c) Give an example of two non-zero vectors that are orthogonal with respect to the inner product in (a).

(5) Let V be the set of all vectors of form (a, b, c) where $a + 2b - c = 0$. Find a basis B for V and find $dim(V)$.

(6) Find a basis for the orthogonal complement of the subspace of $\mathbb{R}^3$ spanned by the vectors $\mathbf{v_1} = (1, -1, 3)$, $\mathbf{v_2} = (5, -4, -4)$, $\mathbf{v_1} = (7, -6, 2)$.

(7) Let V be an inner product space. Prove that if $\mathbf{u}$ is orthogonal to $\mathbf{v_1}, \mathbf{v_2}, \cdots \mathbf{v_n}$, then it is orthogonal to every vector in $span\{\mathbf{v_1}, \mathbf{v_2}, \cdots \mathbf{v_n}\}$.

(8) Let $V = \mathbb{R}^n$ and A be a fixed invertible $n \times n$ matrix with real entries. For $\mathbf{u},\ \mathbf{v} \in \mathbb{R}^n$, interpreted as column matrices, let

$$\langle \mathbf{u}, \mathbf{v} \rangle = A\mathbf{u} \cdot A\mathbf{v},$$

i.e., taking the dot product of $A\mathbf{u}$ with $A\mathbf{v}$. Prove that $\langle \mathbf{u}, \mathbf{v} \rangle$ is an inner product on $\mathbb{R}^n$.

Hint: Note that $\quad A\mathbf{u} \cdot A\mathbf{v} = (A\mathbf{u})^T A\mathbf{v} = (A\mathbf{v})^T A\mathbf{u}$.

(9) Let $V = \mathbb{R}^3$ and $A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 3 & -1 \\ 0 & 0 & 2 \end{bmatrix}$. Define $\langle \mathbf{u}, \mathbf{v} \rangle = A\mathbf{u} \cdot A\mathbf{v}$ as above.
For $\mathbf{u} = (1, -1, 4)$ and $\mathbf{v} = (2, 0, 1)$,
(a) Compute $\langle \mathbf{u}, \mathbf{v} \rangle$.
(b) Compute $d(\mathbf{u}, \mathbf{v})$.
(c) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

(10) Let $S = \{(1, 0, 0),\ (-1, 0, 1), (0, 1, -1)\}$ and $T = \{(1, 1, 0),\ (0, 1, 1),\ (0, 0, 1)\}$ be bases for $\mathbb{R}^3$.
(a) Find the transition matrix $P_{T \leftarrow S}$ from S to T.
(b) Find the transition matrix $P_{S \leftarrow T}$ from T to S.
(c) Let $\mathbf{v} = (1, -2, 3)$. Find $[\mathbf{v}]_S$.
(d) Let $\mathbf{v} = (1, -2, 3)$. Find $[\mathbf{v}]_T$ by using transition matrix found in previous part.

(11) Let $\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ Find the eigenvalues and bases for corresponding eigenspaces of the following matrices:
(a) A.
(b) A^T
(c) A^6