Definitions: A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is said to be *one-to-one* if T maps distinct vectors in $\mathbb{R}^n$ to distinct vectors in $\mathbb{R}^m$.

We can interprete this definition two ways:

- For any vector $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, we have

$$\mathbf{u} \neq \mathbf{v} \implies T(\mathbf{u}) \neq T(\mathbf{v})$$

or equivalently,

$$T(\mathbf{u}) = T(\mathbf{v}) \implies \mathbf{u} = \mathbf{v}$$

- For any $\mathbf{w}$ in $\mathbb{R}^m$, there is exactly one $\mathbf{x} \in \mathbb{R}^n$ such that $T(\mathbf{x}) = \mathbf{w}$.

Equivalent statements: Let $n \times n$ matrix A be standard matrix for linear operator $T : \mathbb{R}^n \to \mathbb{R}^n$. The following statements are equivalent:

- A is invertible
- The range of T is $\mathbb{R}^n$
- T is one-to-one.

Inverse operator: If $T : \mathbb{R}^n \to \mathbb{R}^n$ is a one-to-one linear operator, then inverse operator of T exists and is denoted by T^{-1}.

If A is the standard matrix for T, then A^{-1} is the standard matrix for T^{-1}.

Eigenvalue/Eigenvector: If $T : \mathbb{R}^n \to \mathbb{R}^n$ is a linear operator, then scalar λ is called an *eigenvalue* of T if there is a nonzero vector $\mathbf{x}$ in $\mathbb{R}^n$ such that

$$T(\mathbf{x}) = \lambda \mathbf{x}$$

The nonzero vectors $\mathbf{x}$ are called *eigenvectors* of T.

Note: We just want to have a preliminary idea about eigenvalue/eigenvectors at this point. We will have in depth discussion about this topic in chapter 7.

Linear transformations and polynomials: Let $p(x)$ be an n^{th} degree polynomial.

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x^1 + a_o x^0.$$

Note that when we multiply $p(x)$ by a scalar, $kp(x)$, we just multiply the coefficients by k. Also, when we add two polynomials, we just add corresponding coefficients. Since we only deal with coefficients, associating a polynomial with the vector consisting of its coefficients may be useful.

Let P_n be the set of all polynomials of degree n or less. Then

$$\begin{aligned} T : P_n &\to \mathbb{R}^{n+1} \\ p(x) &\mapsto (a_n, a_{n-1}, a_{n-2}, \cdots a_1, a_o) \end{aligned}$$