MATH 521A – Abstract Algebra
Fall Semester 2014

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Class Days / Time: M, W / 5:30 – 6:45 PM
Location: GMCS-313 (Geology, Mathematics, and Computer Science Building), San Diego State University
Class Website: https://blackboard.sdsu.edu

Office Hours: M, W: 3:00 – 4:00 PM, T: 4:00 – 5:00 PM. If your schedule does not allow you to come to my office during these hours, by all means please make an appointment with me so we can find a mutually agreeable day and time. I *strongly* encourage you to see me if there is anything related to the course that you are unclear on or would like to know more about. I want to help you learn the material and do well in the class.

Textbook

T. W. Hungerford, *Abstract Algebra, An Introduction.* Third Edition, Brooks Cole, 2012.

Course Description, Relevance, and Learning Outcomes

Math 521A is an introduction to abstract algebra. Historically, algebra has been about solving equations. In high school algebra, most of the equations have real variables, that is, variables whose values are assumed to be real numbers. This is rather restrictive for there are many other number systems out there. In abstract algebra, we try to solve equations where the variables come from *different* number systems. Much of this class consists of asking "what other number systems are out there?"

We will start with a fundamental concept in elementary number theory, namely, *divisibility*: the integer b divides the integer a if we can find an integer c such that $a = bc$. This simple concept gives rise to a beautiful theory, encryptions schemes which are used on any computer today, and many famous open problems in mathematics, among other things. From there, we will move on to rings, polynomial rings, ideals, and groups. Basically, the topics covered will include the first seven chapters of the textbook.

Relevance: The teaching of algebra is the largest component of the job of secondary school math teachers. Therefore, the class is useful if you intend on becoming one of them. The course is a necessity if you intend to seek a graduate degree in any of the

branches of mathematics (algebra, geometry, analysis, and logic) or applied mathematics. Besides that, abstract algebra plays an important role in areas such as: engineering (information theory (the design of efficient error-correcting codes) and cryptography (creating and breaking secret messages)), computer science (automata (the development of finite-state machines, for example telephones and vending machines), computer graphics: 3–D rotations, movement of figures, etc.), and physics (the state of sub-atomic particles).

Learning Outcomes: The course will give you insight into a branch of pure mathematics that has both historical and practical significance. By the end of it, you should feel confident to tackle some relevant number-theoretic problems, be able to write a concise algebra proof, be able to read, understand, and criticize basic technical proofs, and describe the fundamental theorems arising from the concepts covered in the course. More specifically, you will:

1. Understand the concepts of congruence and modular arithmetic, including how to solve modular linear equations using the Euclidean algorithm;
2. Read, interpret, and use the terminology, symbolism, and basic definitions from abstract algebra, including binary operations, rings, ideals, fields, integral domains, homomorphisms, groups, and subgroups;
3. Understand arithmetic in polynomial rings, including divisibility and unique factorization;
4. Perform modular arithmetic with polynomials;
5. Be introduced to the concept of ideals and quotient rings; then prove the related fundamental theorems such as the first isomorphism theorem for rings;
6. Be able to prove basic theorems in group theory such as Lagrange's theorem;
7. Use the facts, formulas, and techniques learned in this course to prove theorems about the structure, size, and nature of rings, subrings, ideals, quotient rings, and the associated mappings; order of a group element;

Finally, you will acquire a level of proficiency in the fundamental concepts and applications necessary for further study, including graduate work, in academic areas requiring abstract algebra as a prerequisite, or for work in occupational fields requiring a background in abstract algebra or other highly abstract mathematics. These fields might include the physical sciences and engineering as well as mathematics.

Prerequisite

Discrete Mathematics (MATH 245) and knowledge of matrix algebra, acquired from Linear Algebra (MATH 254).

Exams and Grading

There will be four exams and all of them will be held in class. The final exam will be comprehensive, with an emphasis on the material covered after the third exam:

- **Midterm Exam 1**: Wednesday, September 24. Weight: 250 points.
- **Midterm Exam 2**: Wednesday, October 22. Weight: 250 points.
- **Midterm Exam 3**: Wednesday, November 19. Weight: 250 points.
- **Final Exam:** Friday, December 12, **3:30 – 5:30 PM**. Weight: 250 points. Please note: *The schedule of final exams is determined by the university, and not by the instructor*.

The numerical points for letter grades (A, A–, B+, … ,) will be based only on exam scores. A grade of A is above 85%, A– is above 80%, B is above 70%, C is above 60%, etc. Exams are to be completed alone without the use of calculators, laptops, phones, notes, or assistance from others.

Advice for Doing Well in the Course

Suggested weekly homework problems will be posted on the class website. *Success in the course is correlated with regular attendance and punctual completion of homework.*

Read the book carefully and do the exercises proposed therein. The answers to selected exercises can be found at the end of the book. Doing the exercises as you read the text will clarify the concepts and greatly help your understanding of the material. Experimentation is fundamental for you to understand the material, see patterns, and obtain results. Feel free to work with your colleagues on the homework assignments, but write up your own solutions. *Doing the homework problems yourself is essential in order to do well in the course.*

Students with Disabilities

If you are a student with a disability and believe you will need accommodations for this class, it is your responsibility to contact Student Disability Services at (619) 594-6473. To avoid any delay in the receipt of your accommodations, you should contact Student Disability Services as soon as possible. Please note that accommodations are not retroactive, and that I cannot provide accommodations based upon disability until I have received an accommodation letter from Student Disability Services. Your cooperation is appreciated. Finally, please notify me as well as soon as possible.

References for Further Reading

1. L. N. Childs. A Concrete Introduction to Higher Algebra. Second Edition, Springer, 2001. (very nice introduction).. S. Dummit and R. M. Foote, *Abstract Algebra*, Third Edition. Wiley 2004, 1984.

2. D. S. Dummit and R. M. Foote, Abstract Algebra, Third Edition. Wiley 2004, 1984. (an excellent and complete in-depth introduction).

3. J. B. Fraleigh, A First Course in Abstract Algebra. Seventh Edition. Addison Wesley, 2003. (intermediate).

4. J. Gilbert and L. Gilbert, Elements of Modern Algebra. Sixth Edition. Brooks Cole, 2005. (very nice introduction).

5. I. N. Herstein, Abstract Algebra. Third Edition, Wiley, 1996. (intermediate).

6. T. W. Hungerford, Algebra. Springer-Verlag, Graduate Texts in Mathematics Vol. 73, 1974. (advanced).

7. S. Lang, *Algebra*, Revised Third Edition. Springer-Verlag, Graduate Texts in Mathematics, Vol. 211, 2003. (classic reference).