PRACTICE PROBLEMS FOR EXAM 1

Exam 1 is on September 24th. It will be a closed-book exam covering Chapters 1 and 2 of the textbook. Only pens, pencils, and erasers will be allowed. Calculators, laptops, and phones, will not be allowed.

The proposed practice problems below are those in the textbook, T. W. Hungerford, *Abstract Algebra, An Introduction.* **Third Edition**, Brooks Cole, 2012.

(1) Page 9: 8, 9, 10.

(2) Pages 14–16: 4, 11, 14, 15, 17, 18, 22, 26.

(3) Pages 22–24: 5, 6, 7, 10, 11, 16, 17, 20, 25.

(4) Consider the 13 congruence classes modulo 13, namely, $[0], [1], \ldots, [12]$. For each of the following integers, determine to which class it belongs:

(a) 34, (b) -34, (c) 7, (d) -1, (e) 170, (f) -170.

(5) (a) Evaluate $[1][2][3][4]$ in $\mathbb{Z}_5$.
(b) Using congruences, find the remainder when $4 \cdot 9 \cdot 15 \cdot 59$ is divided by 7.
(c) Find all solutions x to the equation $x^3 + x^2 + x + 1 = 0$ in $\mathbb{Z}_2$. Do the same in $\mathbb{Z}_3$ and $\mathbb{Z}_5$.

(6) Using the Euclidean algorithm, find all solutions x (if they exist) to each of the following equations:
(a) $6x = 33$ in $\mathbb{Z}_{27}$.
(b) $68x = 37$ in $\mathbb{Z}_{40}$.
(c) $2x + 1 = 36$ in $\mathbb{Z}_{65}$.

(7) Show that when a perfect square is divided by 8, the remainder is 0 or 1 or 4. A perfect square is a number of the form n^2 with $n \in \mathbb{Z}$.

(8) (a) Using congruences, develop a test for divisibility by 8.
(b) Using congruences, develop a test for divisibility by 11.
(c) To determine whether a number is divisible by 7, take the last digit off the number, double it, and subtract the doubled number from the remaining number. If the result is divisible by 7, then the original number is divisible by 7; otherwise, the original number is not divisible by 7. This process may need to be applied multiple times in succession. Using congruences, can you explain why this test works?

Example: Is 1715 divisible by 7? Calculate $171 - 2 \cdot 5 = 161$. If you already know the answer, you may stop here, else calculate $16 - 2 \cdot 1 = 14$. The latter is a multiple of 7, hence 1715 is divisible by 7.