

<p>For any random variable X:</p> $E(X) = \sum X_i \cdot p_i$ $Var(X) = \sum (X_i - E(X))^2 p_i$	<p>If X and Y are independent, for any constant a:</p> $E(aX) = a E(X)$ $E(X+Y) = E(X) + E(Y)$ $Var(aX) = a^2 Var(X)$ $Var(X+Y) = Var(X) + Var(Y)$
<p>If $X \sim B(n,p)$</p> $E(X) = np$ $St.Dev.(X) = \sqrt{npq}$ $P(X=r) = \binom{n}{r} p^r q^{n-r}$	

1. In a random sample of 70 adults, 15 had watched 0 movies in the last month, 25 watched 1 movie in the last month, 20 watched 2 movies, and the rest watched 3 movies. Let X be a random variable representing the number of movies a randomly selected adult has watched in the last month.

- Using this data, find the estimated probability distribution of X. (You may round to 2 decimal places during all calculations)

X	0	1	2	3
P(X)	0.21	0.36	0.29	0.14

$$P(X=0) = \frac{15}{70} = 0.21 \quad P(X=1) = \frac{25}{70} = 0.36$$

$$P(X=2) = \frac{20}{70} = 0.29 \quad P(X=3) = \frac{10}{70} = 0.14$$

- Find the expected number of movies a randomly selected adult has watched in the last month

$$E(X) = \sum X_i p_i = 0(0.21) + 1(0.36) + 2(0.29) + 3(0.14)$$

$$= \boxed{1.36}$$

- Find the variance of the number of movies a randomly selected adult has watched in the last month

$$Var(X) = \sum (X_i - E(X))^2 p_i = (0-1.36)^2(0.21) + (1-1.36)^2(0.36) + (2-1.36)^2(0.29)$$

$$+ (3-1.36)^2(0.14) = \boxed{0.93}$$

- Find the standard deviation of the number of movies a randomly selected adult has watched in the last month

$$St.Dev(X) = \sqrt{Var(X)} = \sqrt{0.93} = \boxed{0.96}$$

2. 65% of adults support Proposition B. I randomly select 5 adults. Let X be the number of adults in my sample that support Proposition B

- Describe the probability distribution of X

$$X \sim B(5, 0.65)$$

- How many adults in your sample would you expect to support Proposition B?

$$E(X) = np = 5(0.65) = \boxed{3.25}$$

- What is the standard deviation of the number of adults in your sample that support Proposition B?

$$St\ Dev(X) = \sqrt{npq} = \sqrt{5(0.65)(0.35)} = \boxed{1.07}$$

- Fill in the table below, rounding to 2 decimal places

X	0	1	2	3	4	5
P(X)	0.01	0.05	0.10	0.34	0.31	0.12

$${}^5C_0 (0.65)^0 (0.35)^5 = 0.01$$

$${}^5C_4 (0.65)^4 (0.35)^1 = 0.31$$

$${}^5C_1 (0.65)^1 (0.35)^4 = 0.05$$

$${}^5C_5 (0.65)^5 (0.35)^0 = 0.12$$

$${}^5C_2 (0.65)^2 (0.35)^3 = 0.10$$

$${}^5C_3 (0.65)^3 (0.35)^2 = 0.34$$

- What is the probability at least 1 of the adults in your sample support Proposition B? (You do not need to recalculate, you may use values from the above table).

$$P(X \geq 1) = 1 - [P(X=0)] = 1 - 0.01 = \boxed{0.99}$$

- What is the probability at most 4 of the adults in your sample support Proposition B? (You do not need to recalculate, you may use values from the above table).

$$P(X \leq 4) = 1 - [P(X=5)] = 1 - 0.12 = \boxed{0.88}$$

3. The expected number of words in a text sent by Sally is 25, with a variance of 8. The expected number of words in a text sent by Jane is 42, with a variance of 10. I randomly select 5 text messages sent by Sally, and 2 text messages sent by Jane.

- What is the expected total number of words in all these texts?

$$E(S) = 25 \quad Var(S) = 8 \quad E(J) = 42 \quad Var(J) = 10$$

$$E(5S + 2J) = 5(25) + 2(42) = \boxed{209}$$

- What is the variance of the total number of words in all these texts?

$$Var(5S + 2J) = 5^2(8) + 2^2(10) = \boxed{240}$$

- What is the standard deviation of the total number of words in all these texts?

$$St\ Dev = \sqrt{240} = \boxed{15.49}$$

4. In a random sample of 90 car owners, 46 had taken their car in to get an oil change once in the past year, 22 had taken their car in to get an oil change twice in the past year, and the rest had taken their car in three times to get an oil change in the last year. Find the standard deviation of the number of times a car owner will take their car in to get an oil change in the past year.

X	1	2	3	$P(X=1) = \frac{46}{90} = 0.51$	$P(X=2) = \frac{22}{90} = 0.24$
P(X)	0.51	0.24	0.24	$P(X=3) = \frac{22}{90} = 0.24$	

$$E(X) = 1(0.51) + 2(0.24) + 3(0.24) = 1.71$$

$$\text{Var}(X) = (1-1.71)^2(0.51) + (2-1.71)^2(0.24) + (3-1.71)^2(0.24)$$

$$= 0.68$$

$$\text{St Dev} = \sqrt{0.68} = \boxed{0.82}$$

5. 10% of adults are left handed. I randomly select 10 adults. $X \sim B(10, 0.10)$

- What is the probability 4 of the adults in my sample are left handed?

$$P(X=4) = {}_{10}C_4 (0.10)^4 (0.90)^{10-4} = \boxed{0.01}$$

- What is the probability at least 2 of the adults in my sample are left handed?

$$P(X \geq 2) = 1 - [P(X=0) + P(X=1)] = 1 - [{}_{10}C_0 (0.10)^0 (0.90)^{10} + {}_{10}C_1 (0.10)^1 (0.90)^{10-1}]$$

$$= 1 - [0.35 + 0.397] = \boxed{0.26}$$

6. If $X \sim B(13, 0.93)$, what is the probability X is at most 10?

$$P(X \leq 10) = 1 - [P(X=11) + P(X=12) + P(X=13)]$$

$$= 1 - [{}_{13}C_{11} (0.93)^{11} (0.07)^{13-11} + {}_{13}C_{12} (0.93)^{12} (0.07)^{13-12} + {}_{13}C_{13} (0.93)^{13} (0.07)^{13-13}]$$

$$= 1 - [0.17 + 0.38 + 0.397] = \boxed{0.06}$$

7. The expected number of calories in a randomly selected apple is 95, with a variance of 4. The expected number of calories in a randomly selected orange is 45, with a variance of 3. The expected number of calories in a randomly selected banana is 105, with a variance of 6. You have 2 apples, 4 oranges, and 3 bananas in a fruit bowl.

- What is the expected total weight?

$$E(2A + 4O + 3B) = 2(95) + 4(45) + 3(105) = \boxed{685}$$

- What is the standard deviation of the total weight?

$$\text{Var}(2A + 4O + 3B) = 2^2(4) + 4^2(3) + 3^2(6) = 116$$

$$\text{St Dev} = \sqrt{116} = \boxed{10.77}$$