HW 7

Due Wednesday, Nov. 5

Type I: Ho true, RTN

Type II: Ho false, FTRN

Name: _____

Red ID: _____

1. 1980, 27% of adults had received music instruction at some point in their life. We would like to know whether the percentage has since decreased. In a sample of 120 adults taken this year, 24 had received music instruction.

a) What would constitute a type I error under the appropriate hypothesis test

In reality, the proportion of adults that have received music instruction is 0.27, but we claim it is less

b) What would constitute a type II error under the appropriate hypothesis test

In reality, the proportion of adults that have received music instruction is less than 0.27, but we do not claim this

c). Conduct the appropriate hypothesis test using $\alpha = 0.10$

$\hat{p} = \frac{24}{120} = 0.2$

I. State the null and alternative hypotheses

$H_0: p = 0.27 \quad H_a: p < 0.27$

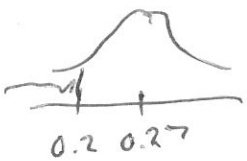
$\hat{p} \sim AN(p, \sqrt{\frac{pq}{n}})$

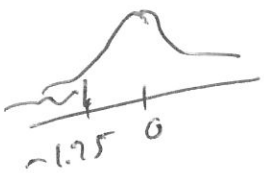
$\hat{p} \sim AN(0.27, \sqrt{\frac{(0.27)(0.73)}{120}})$

$\hat{p} \sim AN(0.27, 0.04)$

II. Find the p-value

$z = \frac{0.2 - 0.27}{0.04} = -1.75$





0.0401

III. Make and justify a decision

0.0401 < 0.10

p-value < α

RTN

IV. Interpret your decision in the context of the problem

There is sufficient evidence to indicate the proportion of adults that have received music instruction has decreased since 1980

d) Interpret your p-value

If the true proportion of adults that have received music instruction really is 0.27, the probability of obtaining a sample proportion of 0.2 or lower is 0.0401.

e) If your decision in your hypotheses test is a mistake, what type of error have you made?

Type I

2. In Alaska the average age of Republicans is 42. We would like to know if the average age of Republicans in Illinois is greater than that of Republicans in Alaska. Assume we know the distribution of the age of all Republicans is normal with a standard deviation of 7 years. We take a sample of 11 Republicans in Illinois, and from this sample calculate an average age of 46.

a) What would constitute a type I error under the appropriate hypothesis test

In reality, mean age of Republicans in Illinois is 42, but we claim it is greater

b) What would constitute a type II error under the appropriate hypothesis test

In reality, mean age of Republicans in Illinois is greater than 42, but we do not claim this

c). Conduct the appropriate hypothesis test using $\alpha = 0.01$

I. State the null and alternative hypotheses

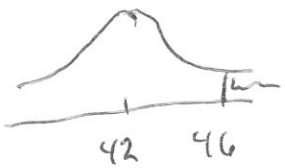
$H_0: \mu = 42 \quad H_a: \mu > 42$

$\bar{X} \sim N(\mu, \sigma/\sqrt{n})$

$\bar{X} \sim N(42, 7/\sqrt{11})$

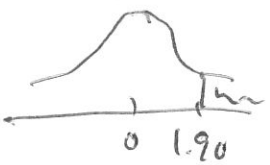
$\bar{X} \sim N(42, 2.11)$

II. Find the p-value



$\bar{X} = 46$

$z = \frac{46-42}{2.11} = 1.90$



$1 - 0.9713 = 0.0287$

III. Make and justify a decision

$0.0287 > 0.01$

p-value $> \alpha$

FTRN

IV. Interpret your decision in the context of the problem

There is not sufficient evidence to indicate the mean age of Republicans in Illinois is greater than that of Republicans in Alaska

d) Interpret your p-value

If the true mean age of Republicans in Illinois is 42, the probability of obtaining a sample mean of 46 or higher is 0.0287.

e) If your decision in your hypotheses test is a mistake, what type of error have you made?

Type II

3. The mean number of hours a 15 year old watches TV a week is 7.2. Assume further that we know the number of hours anyone watches TV per week is normally distributed, with a standard deviation of 3.2 hours. We are interested in whether the mean hours of watching is different for 18 year olds. We take a sample of 4 18 year olds, ask them how many hours they watched TV in the last week, and obtained the following data:

Data: 10 8 13 7 $\bar{X} = 9.5$

a) What would constitute a type I error under the appropriate hypothesis test

In reality, the mean number of hours 18 year olds watch TV is 7.2, but we claim it is different

b) What would constitute a type II error under the appropriate hypothesis test

In reality, the mean number of hours 18 years old watch TV is unequal to 7.2, but we do not claim this

c). Conduct the appropriate hypothesis test using $\alpha = 0.05$

I. State the null and alternative hypotheses

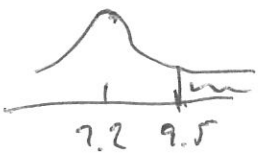
$H_0: \mu = 7.2 \quad H_a: \mu \neq 7.2$

$\bar{X} \sim N(\mu, \sigma/\sqrt{n})$

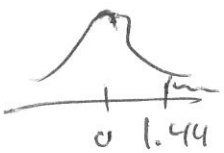
$\bar{X} \sim N(7.2, 3.2/\sqrt{4})$

$\bar{X} \sim N(7.2, 1.6)$

II. Find the p-value



$z = \frac{9.5 - 7.2}{1.6} = 1.44$



$1 - 0.9251$

$= 0.0749$

$0.0749(2)$

$= \boxed{0.1498}$

III. Make and justify a decision

$0.1498 > 0.05$

p-value $> \alpha$

FTRN

IV. Interpret your decision in the context of the problem

There is not sufficient evidence to indicate the mean number of hours 18 year olds watch TV is unequal to 7.2

e) If your decision in your hypotheses test is a mistake, what type of error have you made?

Type II

4. In 2005 82% of registered voters had voted in the last election. We would like to know whether the percentage has since changed. We take a sample of 180 registered voters and find that 135 of them had voted in the last election.

a) What would constitute a type I error under the appropriate hypothesis test

In reality, the percentage of registered voters that voted in the last election currently is 82%, but we claim it has changed since 2005

b) What would constitute a type II error under the appropriate hypothesis test

In reality, the percentage of registered voters that voted in the last election is unequal to 82%, but we do not claim this

c). Conduct the appropriate hypothesis test using $\alpha = 0.05$

I. State the null and alternative hypotheses

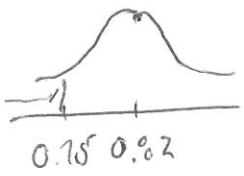
$H_0: p = 0.82 \quad H_a: p \neq 0.82$

$\hat{p} = \frac{135}{180} = 0.75$

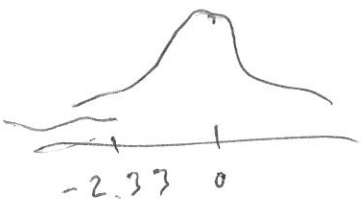
$\hat{p} \sim AN(p, \sqrt{\frac{pq}{n}}) \quad \hat{p} \sim AN(0.82, \sqrt{\frac{(0.82)(0.18)}{180}})$

$\hat{p} \sim AN(0.82, 0.03)$

II. Find the p-value



$z = \frac{0.75 - 0.82}{0.03} = -2.33$



$(0.0099)(2) = 0.0198$

III. Make and justify a decision

$0.0198 < 0.05$

p-value $< \alpha$

RTN

IV. Interpret your decision in the context of the problem

There is sufficient evidence to indicate the percentage of registered voters that voted in the last election has changed since 2005 (is unequal to 82%)

e) If your decision in your hypotheses test is a mistake, what type of error have you made?

Type I