

1. In a random sample of 70 tax returns, 35 claimed 1 dependent, 21 claimed 2 dependents, and the rest claimed 3. Let X be the number of dependents claimed on a randomly selected tax return. What is the standard deviation of X?

- a) 0.29
- b) 0.61
- c) 1.70
- d) 0.78
- e) 1.04

X	1	2	3
P(X)	0.5	0.3	0.2

$$P(X=1) = \frac{35}{70} = 0.5 \quad P(X=3) = \frac{14}{70} = 0.2$$

$$P(X=2) = \frac{21}{70} = 0.3$$

$$E(X) = \sum X_i p_i = 1(0.5) + 2(0.3) + 3(0.2) = 1.7$$

$$Var(X) = \sum (X_i - E(X))^2 p_i = (1-1.7)^2(0.5) + (2-1.7)^2(0.3) + (3-1.7)^2(0.2) = 0.61$$

$$St\ Dev = \sqrt{Var} = \sqrt{0.61} = 0.78$$

Use the following to answer questions 2 and 3

The expected number of emails John will receive in a day is 9.4, with a variance of 3.2. The expected number of emails Aaron will receive in a day is 4.0, with a variance of 1.9. The expected number of emails Susan will receive in a day is 13.0, with a variance of 5.2. We collect all the emails received by John over a period of 3 days, all the emails received by Aaron over the period of 2 days, and all the emails received by Susan over a period of a single day.

$$E(J) = 9.4 \quad Var(J) = 3.2$$

$$E(A) = 4.0 \quad Var(A) = 1.9$$

$$E(S) = 13.0 \quad Var(S) = 5.2$$

2. What is the expected total number of emails collected?

- a) 113.60
- b) 49.20
- c) 18.60
- d) 26.40
- e) 41.60

$$E(3J + 2A + S) = 3(9.4) + 2(4.0) + 13.0 = 49.2$$

3. What is the standard deviation of the total number of emails collected?

- a) 10.66
- b) 41.60
- c) 18.60
- d) 6.45
- e) 4.31

$$Var(3J + 2A + S) = 3^2(3.2) + 2^2(1.9) + 5.2 = 41.6$$

$$St\ Dev = \sqrt{41.6} = 6.45$$

Use the following to answer questions 4 through 7

$$X \sim B(7, 0.842)$$

According to the CDC, 84.2% of adults have received a tetanus vaccination. We randomly select 7 adults.

4. What is the expected number of adults in our sample that have received a tetanus vaccination?

- a) 0.97
- b) 5.89
- c) 4.96
- d) 1.10
- e) 3.79

$$E(X) = np = 7(0.842) = 5.89$$

5. What is the standard deviation of the number of adults in our sample that have received a tetanus vaccination?

- a) 2.42
- b) 5.89
- c) 0.93
- d) 0.97
- e) 1.10

$$St\ Dev(X) = \sqrt{npq} = \sqrt{7(0.842)(0.158)} \approx 0.97$$

6. What is the probability 5 of the adults in our sample have received a tetanus vaccination?

- a) 0.71
- b) 0.92
- c) 0.14
- d) 0.05
- e) 0.22

$$P(X=5) = \binom{7}{5} (0.842)^5 (0.158)^2 = 0.22$$

7. What is the probability at most 5 of the adults in our sample have received a tetanus vaccination?

- a) 0.31
- b) 0.22
- c) 0.78
- d) 0.08
- e) 0.38

$$1 - [P(X=6) + P(X=7)] = 1 - \left[\binom{7}{6} (0.842)^6 (0.158)^1 + \binom{7}{7} (0.842)^7 (0.158)^0 \right] = 1 - [0.39 + 0.3] = 0.31$$

8. If $X \sim B(11, 0.11)$, what is the probability X is at least 3?

- a) 0.11
- b) 0.91
- c) 0.03
- d) 0.35
- e) 0.09

$$1 - [P(X=0) + P(X=1) + P(X=2)] = 1 - \left[\binom{11}{0} (0.11)^0 (0.89)^{11} + \binom{11}{1} (0.11)^1 (0.89)^{10} + \binom{11}{2} (0.11)^2 (0.89)^9 \right] = 1 - [0.28 + 0.38 + 0.23] = 0.11$$