

M337 ELEMENTARY DIFFERENTIAL EQUATIONS

Homework – Series Solutions to ODEs

1. Determine all the singular points of the following differential equations.

(a) $(x + 1)y'' - x^2y' + 3y = 0$.

(b) $(x^2 + x)y'' + 3y' - 6xy = 0$.

(c) $(t^2 - t - 2)x'' + (t + 1)x' - (t - 2)x = 0$.

2. Find at least the first four nonzero terms in a power series expansion about $x = 0$ for a solution to the given differential equation:

(a) $y' + (x + 2)y = 0$.

(b) $y'' - xy' + 4y = 0$.

(c) $(x + 1)y'' - y = 0$, $y(0) = 0$, $y'(0) = 1$.

3. Classify each singular point (real or complex) of the given equation as regular or irregular.

(a) $(x^2 - 1)y'' + xy' + 3y = 0$.

(b) $x^2y'' + 8xy' - 3xy = 0$.

4. Use the method of Frobenius to find at least the first four nonzero terms in the series expansion about $x = 0$ for a solution to the given differential equation for $x > 0$.

(a) $9x^2y'' + 9x^2y' + 2y = 0$.

(b) $x^2y'' + xy' + x^2y = 0$.